**Resumo**— Este trabalho apresenta um modelamento genérico para IMU (Inertial Measurement Unity). Através de ajustes de parâmetros de desempenho de sensor tais como bias, ruído e resposta em frequência além de outros, o modelamento oferece capacidade para simular desempenho de sensor desejado, possibilitando testar requisitos de sensor e desempenho. O modelo é comparado com uma simulação de sensor IMU provida pelo Instituto ZARM da Universidade de Bremen. Os dados de simulação de trajetória são referentes ao protótipo PHOENIX-EADS prototype.

**Palavras-Chave**— IMU, Simulação, Sensor de navegação inercial.

**Abstract**— In this work a generic modeling for IMU (Inertial Measurement Unity) is presented. By setting sensor performance parameters such as bias, noise and frequency response among others, the modeling offers the capability to simulate required sensor performance enabling testing sensor requirements and performance. The model is compared against IMU sensor simulation provided by ZARM Institute of University of Bremen. The trajectory simulation data is related to the PHOENIX-EADS prototype.

**Keywords**— IMU, Simulation, Inertial Navigation Sensor

**I. DEVICE OVERVIEW**

The Inertial Measurement Unity (IMU) is an integrated sensor unity constituted by three accelerometers and three gyroscopes mounted in three orthogonal axes capable of providing measurements of three-dimensional specific force and attitude rate with respect to the inertial reference frame. These measurements, when integrated in time allow to find navigation solution for inertial navigation systems. However, since sensors are not perfect, their measurements are affected by various errors and then affecting navigation solution computations. That is the reason why the knowledge about sensor performance and its internals is important for determining its impacts on a system.

**II. INERTIAL MEASUREMENT UNITY MODEL**

Since there are a lot of types of accelerometers and gyro, the IMU can be assembled in many different ways using different integration strategies and sensor types. This results on different device performances, which can nonetheless be modeled by a common mathematical model by adjusting parameters ([3], [4], [5], [6] and [9]). To ease, the model is analyzed in 3 different levels:

- **Input Change** - the process by which inputs are changed and generate the input actually viewed by the sensor.
- **Sensor Dynamic** - the process by which inputs viewed by the sensor are modified by the sensor dynamic.
- **Measurement** - the process by which the resulting inputs are translated to the sensor output (Displacements or Volts instead of force or attitude rates).

Besides that, some model parameters are modeled in 3 parts as follow:

- **Nominal** - regarded as the mean value, sometimes function of temperature
- **Stability day-day** - regarded as the standard deviation that change the Nominal value from operation to operation, but remaining fixed during one operation time.
- **Stability in run** - regarded as the standard deviation that change the Nominal value during the operation time.

**III. ACCELEROMETER TRIAD GENERIC MODEL**

**A. Gravitational gradient effect**

When the sensor is not under the calibration reference conditions a gravitational force difference effect appears in the specific force \( f_p \) as function of the inertial position \( r_i^p \), attitude \( q_i^p \) and references \( g_0 \) and \( g_0^p \).

\[
\begin{align*}
   f_G^p &= f_p + \Delta g^p = f_p + (\mathbf{T}_p^0 (q_i^p) \mathbf{g}^p (t^p) - \mathbf{T}_p^0 (q_0^p) \mathbf{g}^p (t_0^p)) \\
   \end{align*}
\]

(1)

**B. Input Change**

In the accelerometer, the main sources of signal change are Misalignments, Cross-Coupling and Rotation Sensitivity.

1) **Misalignment** - The misalignment consist of a non-orthogonal transformation represented by the accelerometer cluster misalignment matrix \( \mathbf{T}_M^0 (\alpha (t)) \) with respect to platform reference axes causing that the specific force components are not aligned to each input axis:

\[
\begin{align*}
   f_{M}^p &= \mathbf{T}_M^0 (\alpha (t))^{-1} \cdot f_p \\
   \end{align*}
\]

(2)
\[
\begin{align*}
\delta_D & \text{ term is the stability day-to-day applied to the random values } \gamma_i \text{ and } \delta_R \text{ term is stability-in-run applied to the random values } \mu_i(t). \text{ The nominal value is } \alpha_N. \\
\text{The standard deviations are function of the nominal values } \alpha \text{ and given parameters for stability day-to-day } \alpha_D \text{ and in-run } \alpha_R:  \\
\sigma_D \alpha &= \alpha_D \cdot \alpha  \\
\sigma_R \alpha &= \alpha_R \cdot \alpha
\end{align*}
\]

2) Cross Coupling: The cross coupling effect is due the cross influence of orthogonal components to the input of a given axis regarded as a scaling matrix

\[
\begin{align*}
\vec{f}_n^a &= \vec{f} + K_{\text{NC}}(\vec{x}_c^a(t)) \vec{f}_p^a  \\
K_{\text{NC}}(\vec{x}_c^a(t)) &= \begin{bmatrix}
0  & \vec{x}_{cy}^a & \vec{x}_{cz}^a \\
\vec{x}_{cy}^a & 0  & \vec{x}_{cz}^a \\
\vec{x}_{cz}^a & \vec{x}_{cy}^a & 0
\end{bmatrix}
\end{align*}
\]

where \( \vec{x}_c^a(t) \) are the cross coupling factors modeled similarly as given by equation 4 by using the standard deviations as function of the nominal value \( \vec{x}_c^a \) and given parameters for stability day-to-day \( \vec{x}_{cy}^a \) and in-run \( \vec{x}_{cz}^a \).

3) Rotation Sensitivity: The Rotation Sensitivity effect is due the linear influence of the actuating angular rate \( \omega_{\text{pol}} \) as a scaling matrix to the input of a given accelerometer axis

\[
\begin{align*}
\vec{f}_n^a &= \vec{f} + K_{\text{RS}}(\vec{r}_s^a(t)) (\vec{f}_p^a - \vec{w}_p)  \\
K_{\text{RS}}(\vec{r}_s^a(t)) &= \begin{bmatrix}
\vec{r}_{sx}^a & \vec{r}_{sy}^a & \vec{r}_{sz}^a \\
\vec{r}_{sy}^a & \vec{r}_{sz}^a & \vec{r}_{sx}^a \\
\vec{r}_{sz}^a & \vec{r}_{sx}^a & \vec{r}_{sy}^a
\end{bmatrix}
\end{align*}
\]

where \( \vec{r}_s^a(t) \) are the Rotation Sensitivity factors modeled similarly as given by equation 4 by using the standard deviations as function of the nominal value \( \vec{r}_{sx}^a \) and given parameters for stability day-to-day \( \vec{r}_{sy}^a \) and in-run \( \vec{r}_{sz}^a \).

C. Sensor Dynamic

The main sources of signal change are Frequency Response, Noise, Bias, Sensor Resolution and Input Saturation.
1) Frequency Response: Every sensor as a dynamic system, has its frequency response that affects the spectrum of the signal. To model this effect, an analog filter \( H_a(s) \) can be used where its dynamics is described by its poles \( p_m \), zeros \( z_m \) and gain \( K_a \).

\[
H^a(s) = \frac{\hat{f}_{FR}^a(s)}{f^a(s)} = K^a \cdot \prod_{m=1}^{M_a} \frac{(s + z_m)}{(s + p_m)} \prod_{n=1}^{N_a} (s + p_n)
\]

(12)

However, since the model treats with digital data, this filter must be then translated to a digital filter \( H^a(z) \) ( [1], [10]):

\[
H^a(z) = \frac{\hat{f}_{FR}^a(z)}{f^a(z)} = 1 + \sum_{m=1}^{M_a} b_m^a z^{-m} - \sum_{n=1}^{N_a} c_n^a z^{-n}
\]

(13)

where \( \hat{f}_{FR}^a(z) \) and \( f^a(z) \) are the \( z \) transform of the filter input and output. Note that the \( b_m^a \) coefficients integrate the gain and that the number of analog poles and zeros are not necessarily identical to the number of discrete poles and zeros.

For each accelerometer:

\[
\hat{f}_{FR}^a(k) = \begin{cases} 
\sum_{m=0}^{M} b_m^a \cdot f^a(k - m) + \sum_{n=1}^{N} c_n^a \cdot \hat{f}_{FR}^a(k - n), & k \geq \text{max}(M, N) \\
\hat{f}^a, & k < \text{max}(M, N)
\end{cases}
\]

(14)

where the delayed filter outputs \( \hat{f}_{FR}^a(k - n) \) are put in feedback with the delayed inputs \( f^a(k - m) \) through the digital filter coefficients \( b_m^a \) and \( c_n^a \). Note that at the very beginning, when the number of samples is less then \( \text{max}(M, N) \), there is not enough data to apply the filter. In this case the filter is not applied and the measurement does not suffer any effect.

Let's say that the desired dynamic is modeled by a second order filter:

\[
H^a(s) = K^a \cdot \frac{1}{s^2 + 2\xi_a^a \omega_n^a s + (\omega_n^a)^2}
\]

(15)

Here \((\omega_n^a)^2\) is the cutoff frequency \((\text{rad/s})\) and \(\xi^a\) is the system dumping, which defines at which magnitude the cutoff will take place: \( \text{Mag}_{c}^a = 1/(2 \xi^a) \). This defines a low pass filter and as such it will allow low frequencies while cutting off high frequencies causing a phase delay in the steady state of the signal. Note that the Magnitude response is not normalized. If a normalized response is desired instead, the gain can be adjusted as \( K^a = (\omega_n^a)^2 \).

There is yet an important problem in the discrete domain to be observed. Depending on which is the relation between the speed of dynamic response, translated by system time constant \( \tau^a = 1/(\xi_a \omega_n^a) \) and the system sampling rate \( f_s^a \), that can happen that the system dynamic is much faster than the sampling rate. That means the response of the system is too fast for the sampling process to map enough data to describe the state change of the signal, which means that the application of a digital filtering will not simulate properly the desired dynamic effect.

To illustrate, consider a sampling rate of 100Hz (0.01s). By the Nyquist law sampling the rate should be at least twice faster than the desired changing rate. If the system dynamic response is such that a response happens faster than 0.02s, the change is not detectable and the digital filter will be not able to apply the desired dynamic effect. Although the Nyquist law states the presented relation, in the practice a minimum relation of ten times would be desirable to better describe the dynamic change through samples, what means \( f_s^a \geq 10 \omega_c \).

2) Noise: Noise is regarded as standard deviation due unmodeled internal effects or due non desirable actuating specific forces.

\[
\hat{f}_{Noise}^a = \hat{f}^a + n^a(t)
\]

(16)

\[
n^a(t) = \eta_{EQ}^a \cdot \sqrt{f_s^a} \cdot \mu(t) = \sigma \eta_{EQ}^a \cdot \mu(t)
\]

(17)

where \( \mu(t) \) is a random value variable during the operation, \( f_s^a \) is the sensor sampling rate in \([Hz]\) and \( \eta_{EQ}^a \) is the random walk parameter. Note that it is driven by a standard deviation equivalent random walk, which must be multiplied by the square root of the sampling rate to obtain a real white noise standard deviation.

3) Bias: Bias is the persistent offset of the output with respect to the input due construction imperfections and temperature variations.

\[
\hat{f}_{B}^a = \hat{f}^a + b^a(t, T)
\]

(18)

where \( b^a(t, T) \) is the time variable bias as function of temperature \( T \) modeled similarly as given by equation 4 by using the standard deviations as function of the nominal value \( b^a_\text{N} \) and given parameters for stability day-to-day \( b^a_{27} \) and in-run \( b^a_{12} \). The nominal values \( b^a_\text{N}(T) \) in function of temperature are modeled as order \( n \) temperature polynomial coefficients:

\[
b^a_\text{N}(T) = b^a_0 \cdot T^n + b^a_{n-1} \cdot T^{n-1} + ... + b^a_0
\]

(19)

As an example, the temperature dependency can be modeled as a second order polynomial ( [8]):

\[
b^a(T) = b^a_2 \cdot T^2 + b^a_1 \cdot T + b^a_0
\]

\[
b_{\text{Bias}}\text{std} = \frac{\delta b}{\delta T} \bigg| \text{T=Tstd} = b^a_2 + b^a_1 T_{\text{std}}
\]

\[
b^a_2 = b_{\text{Bias}}\text{std}^a \cdot \frac{b^a(0) - b^a(T_{\text{std}})}{T^2_{\text{std}}}
\]

\[
b^a_1 = 2 b^a(T_{\text{std}}) - b^a(0) - b_{\text{Bias}}\text{std}^a
\]

\[
b^a_0 = b^a(0)
\]

(20)

where the basic values \( b^a(0), b^a(T_{\text{std}}), b_{\text{Bias}}\text{std} \) are given and \( T_{\text{std}} \) represents an adopted standard temperature.

4) Resolution: Resolution \( r_{FR}^a \) is regarded as the minimal value that can be sensed by the sensor represented by means of a quantization process.

\[
\hat{f}_{FR}^a = r_{FR}^a \cdot \text{int} \left( \frac{f^a}{r_{FR}^a} \right)
\]

(21)

where \( \text{int} \) is a function that takes the nearest integer towards zero.
5) Input Saturation: Saturation is regarded as the maximal input value that the sensor can stand, outside of which every input saturates to a maximum value represented by the full scale $s^a$.

$$\tilde{f}_{sat}^a = \begin{cases} f_s^a, & \tilde{f}_a \geq f_s^a \\ \tilde{f}_a, & \tilde{f}_a < f_s^a \end{cases}$$  \hspace{1cm} (22)

D. Measurement

The main sources of signal change are Scale Factor, Scale Factor Non-linearity, Digital Resolution and Delay.

Fig. 5. Accelerometer measurement model.

1) Scale Factor: Scale Factor is represented by a linear gain scaling matrix, which relates the input (specific force) to the output (Displacement or Volts for example).

$$v_{SF}^a(t^a) = K_{SF}^a(t, T) \cdot \tilde{f}_a \cdot \left( \frac{\tilde{f}_a}{f_s^a} \right)^2$$  \hspace{1cm} (23)

$$K_{SF}^a = \begin{bmatrix} s_{f_a}^0 & 0 & 0 \\ 0 & s_{f_y}^0 & 0 \\ 0 & 0 & s_{f_z}^0 \end{bmatrix}$$  \hspace{1cm} (24)

where $s_{f_a}^0$ are the scale factors modeled similarly as for bias given the standard deviations as function of the nominal value $s_{f_a}^N$ as function of the temperature and given parameters for stability day-to-day $s_{f_a}^D$ and in-run $s_{f_a}^R$. $K_{SF}^a(t)$ is the non-linearity in the scale factor treated in next section.

2) Non-linearity: The non-linearity of the scale factor effect is due to the difference of the sensor scale factor from the modeled linear behavior, which cause distortions on the scaling of the input. As shown by equation 23, it can be regarded as a scaling matrix

$$K_{NL}^a = \begin{bmatrix} knl_a^0 & 0 & 0 \\ 0 & knl_a^0 & 0 \\ 0 & 0 & knl_a^z \end{bmatrix}$$  \hspace{1cm} (25)

where the terms $knl_a^*$ are deduced from non-linearity factors $nl^a$ and full scale parameter $f_s^a$ as:

$$v^a(max(f_{in})) = s^f \cdot max(f_{in})$$

$$v^a(max(f_{in})) = s^f \cdot max(f_{in}) + knl^a \cdot max(f_{in})^2$$

$$nl^a = v^a(max(f_{in})) - v^a(max(f_{in}))_{nl}$$

$$f_s^a = max(f_{in})$$

$$knl^a = -\frac{nl^a}{(f_{s}^{a})^2}$$

(26)

Note that the $f_s^a$ parameter must be given in the same unit as the input $f_{in}$ and not in the output unit. However, $nl^a$ is given in the output unit.

The $nl^a$ are the non-linearity modeled similarly as given by equation 4 as function of the nominal value $nl^N_{g}$ and given parameters for stability day-to-day $nl^D_{g}$ and in-run $nl^R_{g}$.

3) Digitization: Digital Resolution $r_D^a$ is regarded as the precision with which the digital output is digitized.

$$\tilde{v}_D^a = r_D^a \cdot \text{int} \left( \frac{\tilde{v}_D^a}{r_D^a} \right)$$  \hspace{1cm} (27)

where $\text{int}$ is a function that takes the nearest integer towards zero.

4) Delay: Delay is the representation of the amount of time that the sensor takes to deliver an output once a given input is presented.

It can be represented by a memory function $f_D^a$ that buffers the current measurement, to only deliver it $\Delta t^a$ seconds later:

$$\tilde{v}_D^a(f_D^a(t), \Delta t^a), \hspace{1cm} t \geq \Delta t^a$$

$$\tilde{v}_D^a(t, \Delta t^a) = 0, \hspace{1cm} t < \Delta t^a$$  \hspace{1cm} (28)

Note that the output is null till the time is greater than $\Delta t^a$.

IV. GYRO TRIAD GENERIC MODEL

The gyros modeling is similar to the one presented for the accelerometers with some differences in the input change part of the model.

A. Input Change

The difference with respect to accelerometers modeling is the absence of rotation sensitivity and the inclusion of G-Sensitivity and G2-Sensitivity.

1) G-Sensitivity: The G-Sensitivity effect is due the linear influence of the actuating specific force as a scaling matrix to the input of a given gyro axis

$$\omega_{GS}^g = \omega_i^g + K_{GS}^g(t) \cdot (T_1^g \cdot T_2^g)$$  \hspace{1cm} (29)

$$K_{GS}^g(g_s^g(t)) = \begin{bmatrix} gs_{xx}^g & gs_{xy}^g & gs_{xz}^g \\ gs_{yx}^g & gs_{yy}^g & gs_{yz}^g \\ gs_{zx}^g & gs_{zy}^g & gs_{zz}^g \end{bmatrix}$$  \hspace{1cm} (30)

where $gs^g(t)$ are the G-Sensitivity factors modeled similarly as given by equation 4 by using the standard deviations as function of the nominal value $gs^N_{g}$ and given parameters for stability day-to-day $gs^D_{g}$ and in-run $gs^R_{g}$.
2) G2-Sensitivity: The G2-Sensitivity effect is due to the square influence of the actuating specific force as scaling matrices to the input of a given gyro axis.

\[
\omega_{G2S}_{ip} = \omega_p^2 + \left( \begin{array}{c} \frac{K_{xG2S}}{g} g_{x2x}(t) \ g_{x2y}(t) \ g_{x2z}(t) \\ \frac{K_{yG2S}}{g} g_{y2y}(t) \ g_{y2z}(t) \\ \frac{K_{zG2S}}{g} g_{z2z}(t) \end{array} \right) \cdot \left( \begin{array}{c} T^{p}_x \\ T^{p}_y \\ T^{p}_z \end{array} \right)
\]

where \( g_{x2x}(t) \) are the G2-Sensitivity factors modeled similarly as given by equation 4 by using the standard deviations as function of the nominal value \( g_{x2x}^0 \) and given parameters for stability day-to-day \( g_{x2x}^0 \) and in-run \( g_{x2x}^0 \).

V. SIMULATION

A. System modeling

The Simulated System is analyzed with inputs from a trajectory simulation provided for Phoenix-EADS while an IMU simulation from ZARM Institute is also provided as comparison.

The first conclusion from the presented pictures is that the proposed model resembles the inputs as expected. It also adds noise and other tendencies to the measured input such as an real sensor. The more clear tendency we can note is that the proposed model allowed the simulation of more realistic bias which has no constant mean over the time.

If the power frequency domain is analyzed through the power spectrum, it can be noted that the proposed IMU modeling does not reshape the base spectrum of the original input while adding some changes around it.

VI. CONCLUSIONS

This paper presented a generic IMU modeling where an example was implemented and compared against IMU modeling simulation provided by ZARM(University of Bremen) for the Phoenix prototype (EADS). The result showed that the proposed model was very similar and resemble the original input despite of sensor disturbances, showing then the model validity. Some features where however noted which present the capability of the model to simulate some behavior of real sensors more accurately than the ZARM-IMU model. The model is then left to the community for further improvement, correction, adaptations as well as verification for other performance parameters.

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REFERENCES

Fig. 7. IMU model and reference trajectory for specific forces.

Fig. 8. Difference between IMU model and reference trajectory for specific forces.

Fig. 9. ZARM IMU model and reference trajectory for specific forces.

Fig. 10. Difference between ZARM IMU model and reference trajectory for specific forces.

Fig. 11. IMU model and reference trajectory for power spectrum specific forces.

Fig. 12. IMU model and reference trajectory power spectrum for angular rates.

Fig. 13. ZARM IMU model and reference trajectory for power spectrum specific forces.

Fig. 14. ZARM IMU model and reference trajectory power spectrum for angular rates.